A STUDY OF LINEAR AND FRACTIONAL MULTI-OBJECTIVE PROGRAMMING PROBLEMS AND ITS APPLICATIONS

SYNOPSIS

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CERTIFICATE

This is to certify that this Synopsis entitled "**A Study of Linear and Fractional Multi-Objective Programming Problems and Its Applications"** by **RACHANA SAINI**, submitted in fulfilment of the requirement for registration in Ph.D. Program in Department of Mathematics under Faculty of Science of JECRC University, Jaipur, during the academic year 2019, is a bonafide record work carried out under our guidance and supervision.

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1. Introduction

Multi-objective programming (also known as multi-objective optimization, vector optimization, multi-criteria optimization, multi-attribute optimization or Pareto optimization) is an area of multiple criteria decision making, that is concerned with mathematical optimization problems involving more than one objective function to be optimized additionally. Optimization problems help to solve involving more than one objective function to be optimized accompanying. In multi-objective analysis, linear fractional objectives are sometimes encountered. Fractional programming concerns with the optimization problem of one or several ratios of functions subject to some linear constraints. These ratios are quantities that measure the efficiency of system. When one or more objectives of a multi-objective programming problem are linear fractional i.e. Ratio of two linear functions under some technological linear restrictions, then the problem is called "multi-objective linear fractional programming (MOLFP) problem". It is a subarea of mathematical optimization. A multiple objective linear programming is a linear program with more than one objective function a MOLP is special case of vector linear programming. MOLP is also a subarea of multi-objective optimization.

When we read about more than one fractional objective function, it is difficult to talk about the optimal solutions of these problems. In the case when several fractional objective functions work, the optimal solution for an objective function may not be optimal solution for some other objective functions. Therefore, we need to find the "best compromise solution", also known as "non dominated solution", "efficient solution", "Pareto optimal solution", "Pareto efficient solution" etc. Thus the concept of optimality in the MOLFP problem is replaced with that of efficiency. A solution is call deficient if none of the objective functions can be improved in value without demeaning the value of any other objective. There may exist a good number of efficient solutions, as vectors cannot be ordered completely, all efficient solutions are equally good.

Linear programming problem

Goerge B. Dantzig (1947) was developed the simplex method of linear programming problem during Second World War. When he and his associates, observed that a large number of military programming and planning problems could be formulated as maximize/minimize a linear form of profit/cost function whose variables were restricted to

values satisfying a system of linear constraints. A linear form is mean a mathematical expression of the type $a_1x_1 + a_2x_2 + \cdots + a_nx_n$, where $a_1 + a_2 + \cdots + a_n$ are constants, and $x_1 + x_2 + \cdots + x_n$ are variables. The term 'programming' refers to the process of determining a particular programme or plan of action. So linear programming has most important optimization techniques in the field of operation research.

Linear programming problem are expressed in mathematical form as

$$\begin{aligned} &Max/Min && Z = C^T x \\ &\text{Subject to} && Ax \leq = \geq b \\ & && x \geq 0 \end{aligned}$$

Where x represents the vector of variables, b is vectors of coefficient, A is a matrix of coefficient, and C^T is the matrix transpose.

Definitions

Feasible Solution - Any set of variables is called a feasible solution of L.P. problem, if it satisfies the set of constraints and non-negativity restrictions also.

Basic Solution – A basic solution to the set of constraints is a solution obtained by setting any n variables (among $m + n$ variables) equal to zero and solving for remaining m variables provided the determinant of the coefficients of these m variables in non-zero. Such m variables are called basic variables and remaining n zero-valued variables are called non-basic variables.

Basic Feasible Solution – A basic feasible solution is a basic solution which also satisfies the non-negativity restrictions, that is, all basic variables are non-negative.

Optimum Basic Feasible Solution - A basic feasible solution is said to be optimum, if it also optimizes (maximizes or minimizes) the objective function.

Pareto Optimal Solution – A solution is called non-dominated or Pareto optimal if none of the objective functions can be improved in value without degrading one or more of the other objective values.

Compromise Optimal Solution - A compromise solution is a solution which is preferred by the decision maker to all other solutions, taking into consideration all criteria contained in the multi-objective functions. Hence, a compromise solution has to be an efficient solution.

Fractional programming

J.R. Isbel and W.H. Marlow (1956) discussed fractional programming. This method is useful in solving the problem in Economics whenever the different economic activities programming utilizes the fixed resources in proportion to the level of their values. Charnes and Cooper (1962) solved the following problem

Fractional programming problem are expressed in mathematical form as

$$Max/Min \qquad Z = \frac{(N^T x + \alpha)}{(D^T x + \beta)}$$

$$\text{Subject to} \qquad Ax \leq = \geq b$$

$$x \geq 0$$

Where x, N and D are $n \times 1$ column vectors, A is an $m \times n$ matrix, b is an $m \times 1$ column vector, The primes (T) over the vectors N and D denote the transpose of vectors, and α, β are some scalars. The constraints have to restrict the feasible region to$\{x : D^T x + \beta \geq 0\}$, i.e. the region on which the denominator is positive.

Multi-objective programming problem

In 1881 at King's College (London) and later at Oxford, economics Professor F.Y. Edgeworth was the first to define an optimum for multi-criteria economic decision making (Edgeworth 1881). He did so for the multi-utility problem within the context of two hypothetical consumer criteria, P and π: "It is required to find a point (x, y) such that in whatever direction we take an infinitely small step, P and π do not increase together but that, while one increases, the other decreases." A multi-objective optimization problem has a number of objective functions which are to be minimized or maximized. Multi-objective optimization problem is following as-

Minimize/Maximize $f_m(x)$ m=1, 2, 3,..., M;

` Subject to $g_j(x) \geq 0$ j=1, 2, 3,…, J;$h_k(x) = 0$ k=1,2,3,...,K;

$$x_i^{(L)} \leq x_i \leq x_i^{(u)} i=1,2,3,\ldots, n$$

Where $f_m(x)$ denotes m objective functions; $g_j(x)$ and $h_k(x)$ are constraint functions; $x_i^{(L)}$ and $x_i^{(u)}$ are respectively a lower and upper bound.

2. Literature review

The study of multi-objective programming problem has been much progress during the few decades in the mathematical programming and its applications to real world public decision making problems. It represents a very useful generalization of more traditional approaches to programming problem. Nykowski (1985) well known multi-objective programming problem. Then he formulated multi-objective programming problem (MOPP) to multi-objective fractional programming (MOFP) and related between their problems are examine. Jain et al. (2010) develop an algorithm for the solution of multi-objective linear plus fractional programming problem (MOLPFPP) when some of the constraints are homogeneous in nature. Guzel et al. (2013) proposed a new solution's theorem is based on multi-objective linear fractional programming problem (MOLFPP) that deals with nonlinear fractional programming with single objective function and studied in the work. Valipour et al. (2014) suggests an iterative parametric approach for solved to multi-objective linear fractional programming (MOLFP) problems which only used linear programming to obtain efficient solutions and always converged to an efficient solution. Chebyshev Goal Programming and Value function approaches are suggested to derive the optimum solution in Multi-objective Linear plus Linear Fractional Programming Problem (MOLPLFPP) presented by Sadia et al. (2015). Lachhwani (2015) proposed modified method numerator and denominator function of each objective function at each level are separately transformed into fuzzy goals and their aspiration levels are determined using separate best solutions. Simi et al. (2017) presented a new approach for solving in which the objective function is a linear fractional function, while constraint functions are in the form of linear inequalities in linear fractional programming problem (LFPP). Here first we transform this LFP problem into linear programming (LP) problem and then solved to this problem algebraically using the concept of duality and compared other methods. Arya et al. (2018) develop an algorithm for generating trade-off pairs between cost and time for quadratic fractional transportation problem with impurity restriction. The algorithm will generate all the solutions which are pareto optimal with respect to transportation cost and the transportation time.

In multi-objective programming problem deals a solution of problem has been formulated using fuzzy set in proposed methodology. Using its approach a procedure has been explored given by Chakraborty et al. (2002). A convergent algorithm based on Stackelberg strategy is employed to solve the (MMILP) problem which does not increase the complexities of the problem considered by Gaur et al. (2008). Abo-sinna et al. (2010) proposed procedure, bi-level multi-objective linear fractional programming problem solved by fuzzy goal programming. It decision variables controlled by first-level decision maker are developed first in the model formulation of the problem.Yap (2014) focus only on non-interactive exact methods that generate the entire set of optimal solutions. Das et al. (2016) constructed a new proposed method in real life problems which is obtained from combination of Charnes−Cooper scheme and the multi-objective linear programming problem. This modified method provides multiple efficient solutions to the problem given by Pramy et al. (2017). Sharma et al. (2017) to find all non-dominated points by an algorithm uses ϵ-scalarization technique and a ranking approach of the integer feasible solution. Osman et al. (2017) presents a fuzzy goal programming approach with fuzzy demands firstly the concept of α-cut and sum of negative deviational variables for solving multi-level multi-objective fractional programming problem. Vandana et al. (2018) introduce the definition of the higher-order $(V, \alpha, \beta, \rho, d)$-invexity. Three duality models for a multi-objective fractional programming problem (MOFPP) involving non differentiability in terms of support functions have been formulated and usual duality relations.

3. Description of topic

A study of linear and fractional multi-objective programming problems and its applications is our topic of study. In which, we will discussing several algorithms, formulation using evolutionary algorithms, fuzzy sets and systems, goal programming and weighted sum approach related to our topic.

4. Objectives of research

The objectives of the proposed study are as follows:

- To study linear and fractional multi-objective programming problems.
- Develop the solving techniques of linear and multi-objective programming problems.
- To study fuzzy theory in linear and multi-objective programming problems.

- To discuss more for less paradox situations in multi-objective transportation problems.
- Case study of multi-objective programming problem in real life.

5. Research methodology to be adopted

We will discover new techniques for solving multi-objective programming problem, where the functions are linear and fractional Evolutionary Algorithms (Weighted Sum Method, ε-Constraint Methods etc.) used to find the new techniques. We will discuss new techniques about using fuzzy sets and systems in linear and fractional multi-objective programming problem.

6. Expected outcome of the research

The proposed study is aimed to give following expected outcomes:

- Study of multi-objective optimization problem with linear and fraction objective function gives very efficient solution to real life problems. It helps us to take good decision.
- Using more for less paradox in multi-objective transportation problem we can ship more goods from origins to destinations with lesser or equal cost.
- Current research will provide knowledge of how multi-objective optimizations are incorporated into management problem solving.
- The case study will demonstrate effective interpretation of multi-objective optimization in real life problems.

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